CBCS SCHEME

USN 4VP20EC082 **18EC44**

Fourth Semester B.E. Degree Examination, July/August 2022
Engineering Statistics and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define an uniform random variable. Obtain the characteristic function of an uniform random variable and using the characteristic function derive its mean and variance. **(08 Marks)**

b. If the probability density function of a random variable is given by

$$f_X(x) = \begin{cases} C\exp(-x/4), & 0 \le x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the value that C must have and evaluate $F_X(0.5)$. **(06 Marks)**

c. The density function of a random variable is given as

$$f_X(x) = a\,e^{-bx} \quad x \ge 0$$

Find the characteristic function and the first two moments. **(06 Marks)**

OR

2 a. Define a Poisson random variable. Obtain the characteristic function of a Poisson random variable and hence find mean and variance using the characteristic function. **(08 Marks)**

b. Suppose 'X' is a general discrete random variable with following probability distribution. Calculate mean and variance for X.

X	0	1	3	5	7
P(X)	0.05	0.2	0.6	0.1	0.05

(06 Marks)

c. The number of defects in a thin copper wire follows Poisson distribution with mean of 2.3 defects per millimeter. Determine the probability of exactly two defects per millimeter of wire. **(06 Marks)**

Module-2

3 a. Define and explain Central Limit theorem and show that the sum of the two independent Gaussian random variables is also Gaussian. **(08 Marks)**

b. Let 'X' and 'Y' be exponentially distributed random variable with

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

Then obtain the characteristic function and Pdf of $W = X + Y$. **(06 Marks)**

c. Determine a constant b such that the given function is a valid joint density function.

$$f_{XY}(x, y) = \begin{cases} b(x^2 + 4y^2) & 0 \le |x| < 1 \text{ and } 0 \le y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

(06 Marks)

OR

4 a. Explain briefly the following random variables :
(i) Chi-square Random Variable
(ii) Rayleigh Random Variable. **(04 Marks)**

b. The joint density function of two random variables X and Y is

$$f_{X,Y}(x,y) = \begin{cases} \dfrac{(x+y)^2}{40}, & -1 < x < 1 \text{ and } -3 < y < 3 \\ 0, & \text{elsewhere} \end{cases}$$

Find (i) the variances of X and Y (ii) the correlation coefficient. **(08 Marks)**

c. Gaussian random variables X_1 and X_2 whose $\overline{X}_1 = 2$, $\sigma^2_{X_1} = 9$, $\overline{X}_2 = -1$, $\sigma^2_{X_2} = 4$ and $C_{X_1X_2} = -3$ are transformed to new random variables Y_1 and Y_2 such that

$$Y_1 = -X_1 + X_2$$
$$Y_2 = -2X_1 - 3X_2$$

Find (i) $\overline{X_1^2}$ (ii) $\overline{X_2^2}$ (iii) $\rho_{X_1X_2}$ (iv) $\sigma^2_{Y_1}$ (v) $\sigma^2_{Y_2}$ (vi) $C_{Y_1Y_2}$ (vii) $\rho_{Y_1Y_2}$ **(08 Marks)**

Module-3

5 a. With the help of an example, define Random process and discuss distribution and density functions of a random process. Mention the differences between Random variable and Random process. **(08 Marks)**

b. Define the Autocorrelation function of the random process X(t) and discuss its properties. **(06 Marks)**

c. A stationary ergodic random process has the autocorrelation function with periodic components as $R_{XX}(\tau) = 25 + \dfrac{4}{1+6\tau^2}$

Find the mean and variance of X(t). **(06 Marks)**

OR

6 a. The autocorrelation function of a wide sense stationary process.

$$R_X(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T}, & -T \le |\tau| \le T \\ 0, & \text{elsewhere} \end{cases}$$

Obtain the Power Spectral Density of the process. **(06 Marks)**

b. Show that the random process $X(t) = A\cos(w_c t + \theta)$ is wide sense stationary. Here θ is uniformly distributed in the range $-\pi$ to π. **(08 Marks)**

c. X(t) and Y(t) are independent, jointly wide sense stationary random processes given by

$$X(t) = A\cos(w_1 t + \theta_1)$$
$$Y(t) = B\cos(w_2 t + \theta_2)$$

If $W(t) = X(t) \cdot Y(t)$ then find the Autocorrelation function $R_W(\tau)$. **(06 Marks)**

Module-4

7 a. Define vector subspaces and explain the four fundamental subspaces. **(06 Marks)**

b. Show that the vectors (1, 2, 1), (2, 1, 0), (1, –1, 2) form a basis of R^3. **(06 Marks)**

c. Apply Gram-Schmidt process to the vectors $v_1 = (2, 2, 1)$, $v_2 = (1, 3, 1)$, $v_3 = (1, 2, 2)$ to obtain an orthonormal basis for $v_3(R)$ with the standard inner product. **(08 Marks)**

OR

8 a. Determine the null space of each of the following matrices:

(i) $A = \begin{bmatrix} 2 & 0 \\ -4 & 10 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -7 \\ -3 & 21 \end{bmatrix}$ **(06 Marks)**

b. Determine whether the vectors (2, –2, 4), (3, –5, 4) and (0, 1, 1) are linearly dependent or independent. **(06 Marks)**

c. Find the QR-decomposition for the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 0 & 7 \\ 0 & -1 & -1 \end{bmatrix}$$

and write the result in the form of $A = QR$. **(08 Marks)**

Module-5

9 a. If $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$

find eigen values and corresponding eigen vectors for matrix A. **(08 Marks)**

b. Diagonalize the following matrix:

$$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$$

Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. **(08 Marks)**

c. What is the positive definite matrix? Mention the methods of testing positive definiteness. **(04 Marks)**

OR

10 a. Factorize the matrix A into $A = U \Sigma V^T$ using SVD.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$$ **(08 Marks)**

b. If $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ show that A is positive definite matrix. **(04 Marks)**

c. Find a matrix P, which transforms the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to diagonal form. **(08 Marks)**

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